ACCURATE AND SIMPLE FORMULAS FOR DISPERSION IN FIN-LINES

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SUMMARY

Very simple formulas are derived describing dispersion in both bilateral and unilateral fin-lines. Their excellent accuracy is shown by comparing with results from spectral domain techniques.

INTRODUCTION

Dispersion in fin-lines has often been dealt with in literature either by applying spectral domain techniques /1-4/ or by constructing approximate but closed-form solutions for the phase velocity of the fundamental mode /5,6/. These formulas can conveniently be applied, their validity is, however, either poor /5/ or limited to small slot widths and special values for the permittivity of the substrate /6/. Another analysis adopting Cohn's technique for slot lines on dielectric substrate has been presented in /7/.

In this paper a simple, analytical method for approximately determining the dispersion of the fundamental mode in bilateral and unilateral fin-lines is given. The procedure is based on the transverse resonance method in conjunction with simple equivalent transverse networks, which take the equivalent susceptance imposed by the fins into account. The computed results are compared with data from /3/, /6/ and /7/.

The analysis is based on the following simplifications:
- isotropic, homogeneous and lossless dielectric layer;
- zero-thickness metallisation with infinite conductivity;
- symmetrically located slot.

Meier has suggested that dispersion in fin-lines may be described by /8/

$$\frac{k_o}{k_z}=\left[k_e-\left(\frac{k_{ca}}{k_o}\right)^2\right]^{-\frac{1}{2}} \qquad (1)$$

and

$$k_e \simeq k_c=\left(\frac{k_{ca}}{k_{cd}}\right)^2 \qquad (2)$$

where

$$k_z=\frac{2\pi}{\lambda_g}\ ;\ k_o=\frac{2\pi}{\lambda_o}\ .$$

λ_g means guide wavelength, λ_o free-space wavelength, k_{cd} cutoff wave number of the dominant mode in the actual fin-line and k_{ca} cutoff wave number in an equivalent air-filled ridged-waveguide substructure of same dimensions. k_c is called equivalent dielectric constant at cutoff and k_e equivalent dielectric constant which may be assumed to be independent of frequency through a waveguide band if thickness and permittivity of the substrate are small.

In the following, our task will be to find the cutoff wave numbers k_{ca} and k_{cd}.

BILATERAL FIN-LINE

The cross section of a bilateral fin-line and its equivalent transverse network at cutoff are shown in Fig. 1. The fin-line is symmetrical with respect to the ($x=\frac{a}{2}$)-plane, where the admittance in x-direction at cutoff is zero. The equivalent transverse network for the dominant mode consists of a capacitive susceptance shunting the TE-mode transmission line with short-circuit termination. The cutoff wave number k_x is determined by the resonance

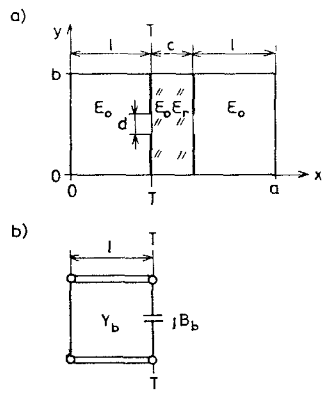

Fig. 1 Cross section of a bilateral fin-line (a) and equivalent transverse network (b)

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condition at reference plane T:

$$-ctg(k_x \cdot l)+\frac{B_b}{Y_b} = 0 \qquad (3)$$

with

$$\frac{B_b}{Y_b} = \frac{b}{\pi}\cdot k_x \cdot (P_w+\varepsilon_r \cdot P_d) \qquad (4)$$

where

$$P_w=\ln(\csc \alpha_w) \qquad (5)$$

$$P_d=r_d \cdot \tan^{-1}(\frac{1}{r_d})+\ln\sqrt{1+r_d^2} \qquad (6)$$

and

$$\alpha_w=\frac{\pi}{2}\cdot\frac{d}{b} \;;\; r_d=\frac{c}{d}.$$

The field distortions by the metal fins have been modelled by the susceptance jB_b which is composed of two parts: The first part models the field distortions left of reference plane T. It can be taken from Marcuvitz' Waveguide Handbook /9/, page 218, where the susceptance of a window of zero thickness in a rectangular waveguide has been derived. (Actually it is sufficient to take just the first term in P_w into account (eq. (5)), because the other terms influence the results by less than 0.2 per cent.) The second part models the field distortions between plane T and the symmetry plane x = a/2. It has been taken from the equivalent circuit of an open E-plane T-junction (see /9/, page 337).

The cutoff wave numbers k_{ca} and k_{cd} are obtained from the first zero of eq. (3) for $\varepsilon_r=1$ and $\varepsilon_r\neq1$, respectively. Numerical results have been compared to those taken from /6/ and an agreement of better than 1 per cent has been found. (Setting $\varepsilon_r=2.22$, $(k_{cd}b)/(2\pi)$ varies between 0.1 and 0.2 for d/b between 1/32 and 3/4. In all cases the deviations are smaller than 0.001.) Fig. 2 shows dispersion in a bilateral fin-line of relatively small slot width. The agreement to results taken from /7/ is very good.

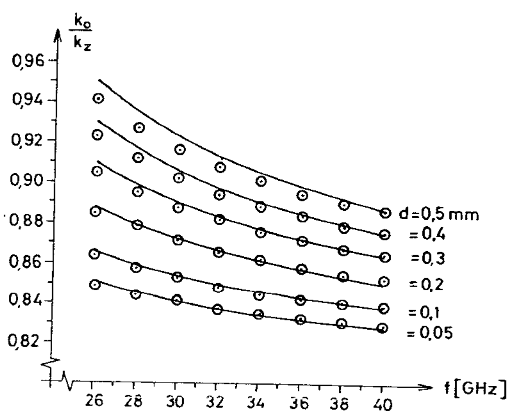

Fig. 2 Dispersion of a bilateral fin-line with slot centered in waveguide
Parameters: $\varepsilon_r=2.22$, a=7.112 mm, b=3.556 mm, c=0.508 mm.

——— results from /7/
⊙ solutions of eq. (3)

UNILATERAL FIN-LINE

Unilateral fin-line is often pref-ered, because its attenuation constant is smaller than that of a bilateral fin-line and because the slot pattern can be defined more easily. The cross section of this fin-line is shown in Fig. 3. The metallization is placed at $(x=\frac{a}{2})$-plane, so that the structure is symmetrical for $\varepsilon_r=1$. It is useful to construct two equivalent transverse networks at cutoff for the dominant mode: one for $\varepsilon_r=1$ and another one for $\varepsilon_r\neq1$ (Fig. 3). The equivalent network for $\varepsilon_r=1$ consists simply of a transmission line of length a/2 which is short-circuited at one end and shunted by jB_w at the other. The shunt susceptance can be taken from eqs. (4) and (5).

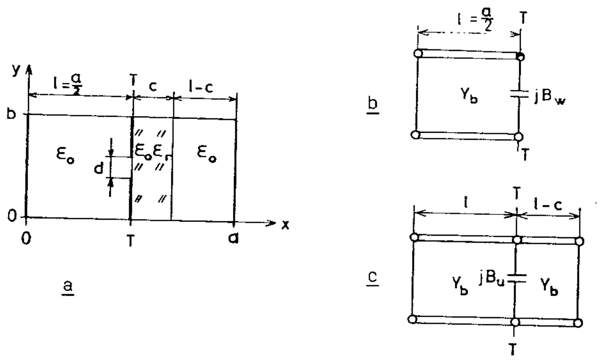

Fig. 3 Cross-section of a unilateral fin-line (a) and equivalent transverse networks for $\varepsilon_r=1$(b) and $\varepsilon_r\neq1$(c)

The cutoff wave number k_{ca} in the air-filled ridge waveguide is determined by the resonance condition at reference plane T:

$$-ctg(k_{ca}\cdot l)+\frac{B_w}{Y_b}=0 \qquad (7)$$

with

$$\frac{B_w}{Y_b}=\frac{b}{\pi}\cdot k_{ca}\cdot P_w. \qquad (8)$$

P_w is given by eq. (5).

The equivalent network in Fig. 3c is formed by susceptance jB_u shunted by two TE-mode transmission lines with short-circuit terminations. The relation governing cutoff of the TE_{10}-mode is given by

$$-ctg(k_{cd}\cdot l)-ctg[k_{cd}\cdot(l-c)]+\frac{B_u}{Y_b}=0 \qquad (9)$$

with

$$\frac{B_u}{Y_b}=\frac{b}{\pi}\cdot k_{cd}\cdot[2P_w+\varepsilon_r(P_d+P_b)] \qquad (10)$$

where

$$P_b = r_b \cdot \tan^{-1}\left(\frac{1}{r_b}\right) + \ln\sqrt{1+r_b^2} \qquad (11)$$

and

$$r_b = \frac{c}{b}.$$

Susceptance jB_u has been constructed by superposing the susceptance of the window (the term proportional to $2P_w$) and another susceptance (the term proportional to $\varepsilon_r(P_d+P_b)$) representing the influence of the dielectric and the transformation through the layer of length c. While P_w (eq. (5)) and P_d (eq. (6)) have already been given, P_b in eq. (11) has been taken from /9/, page 337.

The unilateral fin-line with metal fins centered in waveguide (Fig. 3) has been analyzed in /3/, /6/ and /7/. Other authors considered a unilateral fin-line with dielectric layer symmetrically located in the waveguide /1/, /4/. The metallisation is then at $(x=l_s=\frac{a-c}{2})$-plane. Then the resonance conditions are for $\varepsilon_r=1$

$$-\mathrm{ctg}(k_{ca}\cdot l_s) - \mathrm{ctg}\left[k_{ca}\cdot(l_s+c)\right] + \frac{2\cdot B_w}{Y_b} = 0 \qquad (12)$$

and for $\varepsilon_r \neq 1$

$$-2\mathrm{ctg}(k_{cd}\cdot l_s) + \frac{B_u}{Y_b} = 0 \qquad (13)$$

where $\frac{B_w}{Y_b}$ and $\frac{B_u}{Y_b}$ are given by eq. (8) or (10), respectively.

Comparing the cutoff wave number of the unilateral fin-line to results published in /6/ yields deviations of less than 1.5 per cent if c/a<1/8. This restriction is, however, mostly fulfilled for practical fin-lines. Some dispersion curves are presented in Fig. 4 showing good agreement (better than 3 per cent) with published results calculated with the spectral domain technique.

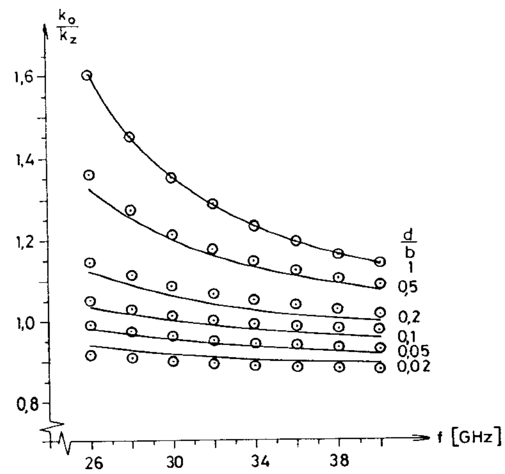

Fig. 4 Dispersion of a unilateral fin-line with slot centered in waveguide.
Parameters: ε_r=2.22, a=7.112 mm, b=3.556 mm, c=0.127 mm.
———— results from /3/
⊙ our approximation

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